Structure in #SAT and QBF

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This thesis considers structural properties that can be used in the design of efficient algorithms for the satisfiability problem of quantified Boolean formulas (QSAT) and propositional model counting (#SAT).

QSAT. The nesting of existential and universal quantifiers in quantified Boolean formulas (QBFs) generates dependencies among variables that have to be respected by QSAT solvers. In standard decision algorithms, it is assumed that all possible variable dependencies exist. Often, only a fraction of these dependencies is realized, while the remaining, "spurious" dependencies lead to unnecessary restrictions that inhibit solver performance. We study dependency schemes as a means to identifying spurious dependencies and establish the following results.

- Among dependency schemes considered in the literature, the resolution-path dependency scheme identifies a maximal set of spurious dependencies. We prove that the resolution-path dependency scheme can be computed in polynomial time.
- We state sufficient conditions for the sound deployment of dependency schemes in search-based QSAT solvers and prove that these conditions are met by several dependency schemes, including those implemented in the solver DepQBF and a variant of the resolution-path dependency scheme.
- We show that known dependency schemes support a reordering operation that is more powerful than quantifier shifting, and present an application to the reduction of quantifier alternations of a QBF.

#SAT. The model counting problem (#SAT) asks for the number of satisfying assignments of a propositional formula in conjunctive normal form. We prove the following results on the complexity of #SAT with respect to structural parameters based on graph width measures, identifying new classes of formulas amenable to efficient model counting.

- We define the modular treewidth of a graph as its treewidth after contraction of modules, and prove that #SAT is polynomial-time tractable for classes of formulas with incidence graphs of bounded modular treewidth.
- Symmetric clique-width is a graph parameter that generalizes treewidth as well as modular treewidth. We show that #SAT is polynomial-time tractable for classes of formulas with incidence graphs of bounded symmetric clique-width.