Presentation of the Diploma Thesis, Faculty of Informatics

Minimal Combinatorial Maps for analyzing 3D Data

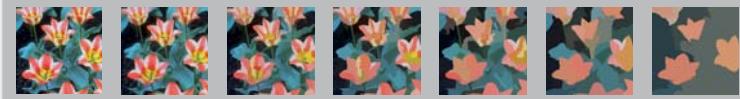
Thomas Illetschko

Computer Science (E881)
Diploma Thesis

Vienna University of Technology
Institute of Computer Aided Automation
Pattern Recognition and Image Processing Group
O.Univ.Prof. Dr. Walter G. Kropatsch

Motivation

Topology preserving hierarchies with Combinatorial Maps are a useful concept in the field of Pattern Recognition and Image Analysis.
Successfull implementations for algorithms based on 2D data exist.



Combinatorial pyramids are defined for any dimension but recent studies focused on 2D data.
One reason is the huge size of initial 3D combinatorial maps.
An initial map encoding 1s of a movie with 25fps of resolution 640x480 uses

~186 million darts.

Goal of the thesis: Define operations to collapse maps to minimal ones while preserving the topology.

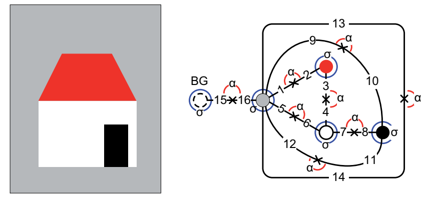
Minimal Combinatorial Maps

A nD Combinatorial map G = (D, β_1, . . . , β_n) is a graph encoding the orientation using darts connected by n permutations.

The cells (vertices, edges, faces, etc.) and topological relations are encoded by these permutations.

In a minimal combinatorial map, each connected component is represented by a single vertex.

$n_v(G) = k + 1$



D: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}
α: {1,2}, {3,4}, {5,6}, {7,8}, {9,10}, {11,12}, {13,14}, {15, 16}
σ: {1,5,12,14,16,13,9}, {2,3}, {4,7,6}, {8,10,11},{15}

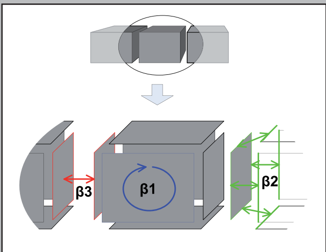
Extending Combinatorial Maps to 3D

3 permutations β_1, β_2, β_3 are used for 3D [1].

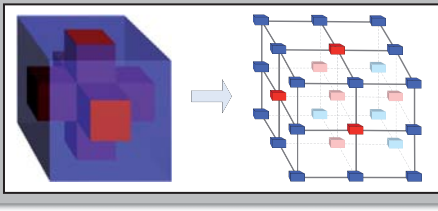
β_1 creates faces by encoding edges around a face.

β_2 creates volumes by connecting faces.

β_3 connects the volumes.



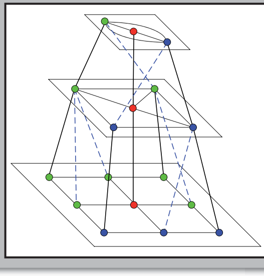
Volumetric data (a stack of 2D images or movies) can be transformed into a grid-like combinatorial map.



Irregular pyramids can be built starting from the initial maps.

Each level is a reduced map of the level below.

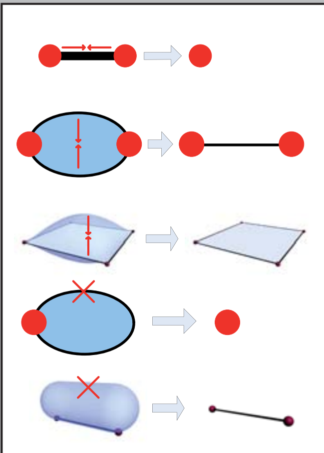
Contraction and removal operations can be used to reduce an initial map preserving the topology. [2]



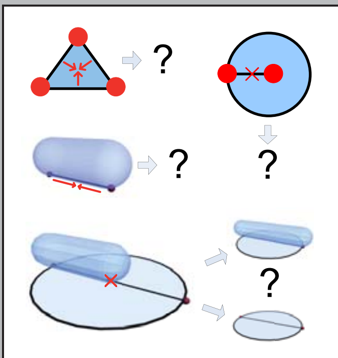
Collapsing 3D Combinatorial Maps

Five operations are used for 3D.

- Edge Contraction merges two adjacent vertices of one connected component.
- Face Contraction collapses a face bounded by parallel edges.
- Volume Contraction reduces a volume bounded by parallel faces.
- Edge Removal eliminates empty edge self-loops.
- Face Removal eliminates empty face self-loops.



10 conditions ensure that the result will be an unambiguous and valid combinatorial map preserving adjacency and inclusions.



It is shown that any 3D combinatorial map that does not contain face self-loops can be reduced to an equivalent minimal 3D combinatorial map using these operations and conditions.

A 3D combinatorial map containing *k* connected components is reduced to exactly ***k + 1*** vertices.
E.g. 1s of a movie with 25fps of resolution 640x480 that shows 10 simple, non-ocluded objects is reduced to a map encoding

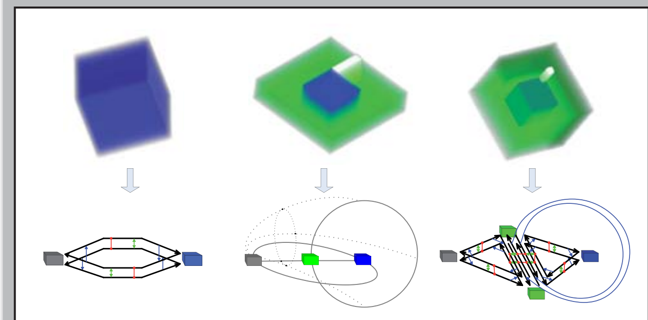
11 vertices.

Cells of a higher dimension can be reduced as well.
Within minimal configurations adjacency, surrounds and contains can be identified using pseudo elements [3].
Thus it is possible to discriminate the 3 primitive topological configurations simplex, tunnel, and hole.

More than one minimal configuration exists for these configurations.

Experimental Results

Representative configurations are reduced.
The final combinatorial maps are defined by the number of objects and their structure.
Pseudo elements allow the identification of topological relations.



A second set of experiments demonstrates the reduction to minimal 3D combinatorial maps.

The initial maps are reduced to minimal combinatorial maps containing ***k + 1*** vertices.

level	darts	vertices	edges	faces	volumes
0	1608672	65601	201204	205725	70122
1	809312	32801	101264	104225	35762
2	409632	16401	51294	53475	18582
3	209792	8201	26309	28100	9992
4	112096	4101	13875	15530	5756
5	63248	2051	7658	9245	3638
6	32176	1026	3934	4856	1948
7	15408	514	1938	2388	964
8	7440	258	948	1170	480
9	3560	130	455	565	240
10	1568	66	204	254	116
11	728	34	94	122	62
12	320	18	43	55	30
13	146	10	20	26	16
14	48	6	9	9	6
15	26	4	5	5	4
16	6	3	3	1	1
17	**4**	**2**	**2**	**1**	**1**

Conclusions

Any 3D Combinatorial map that does not contain a face self-loop can be reduced to an equivalent minimal 3D combinatorial map. In such a map every connected component is encoded by a single vertex.

Topological relations (adjacency, surrounds and contains) can be identified in a minimal 3D configuration using pseudo elements. Thus combinatorial maps can be used for algorithms that require information about the topological structure of the encoded data.

The number of vertices in a minimal 3D combinatorial map is only dependent on the number of connected components. Therefore minimal 3D combinatorial maps have great potential for analyzing 3D data: a reduced size while providing information about the topological relations present in the volumetric data.

References

[1] Braquelaire, A., Damiand, G., Domenger, J.-P., and Vidil, F. (2003). Comparison and convergence of two topological models for 3d image segmentation. In Workshop on Graph-Based Representations in Pattern Recognition, number 2726 in Lecture Notes in Computer Science, pages 59–70, York, England.
[2] Brun, L. and Kropatsch, W. G. (1999a). Dual Contraction of Combinatorial Maps. Technical Report PRIP-TR-54, Institute f. Computer Aided Automation 183/2, Pattern Recognition and Image Processing Group, TU Wien, Austria.
[3] Illetschko, T., Ion, A., Haxhimusa, Y., and Kropatsch, W. G. (2006). Collapsing 3D Combinatorial Maps. In F. Lenzen, O. Scherzer and Vincze, M., editors, Proceedings of 30th Austrian Association for Pattern Recognition Workshop, pages 85–93. Oesterreichische Computer Gesellschaft.

Contact

Thomas Illetschko
thomas.illetschko@ucs.at

O.Univ.Prof. Dr. Walter G. Kropatsch
krw@prip.tuwien.ac.at