FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

Diplomarbeitspräsentation

Logics for Coalitional Games of Imperfect Information

Masterstudium:
Computational Intelligence

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Motivation: A logic for coalitional games of perfect information

Coalitional game: Players are allowed to form coalitions.

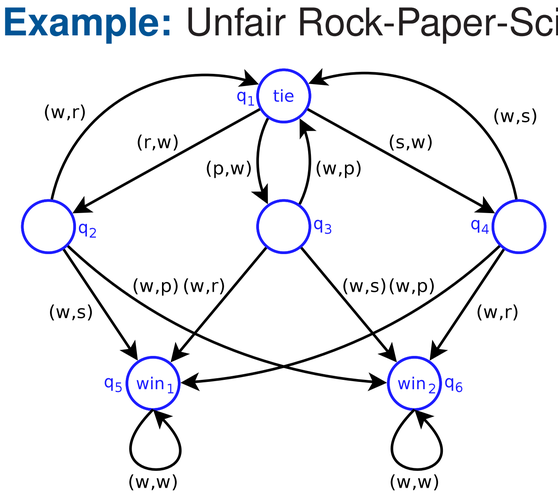
Question: What can those coalitions achieve?
⇒ Abilities of players can be formalized using **modal logics**.

Well-established logic for perfect information games:
Alternating-Time Temporal Logic (ATL), with modalities

- $\langle\langle A \rangle\rangle$ ... coalition A can achieve,
- ◯ ... in the next state of the game,
- □ ... always,
- $\mathcal{U}$ ... until.

Game model: Concurrent game structures

Example: Unfair Rock-Paper-Scissors



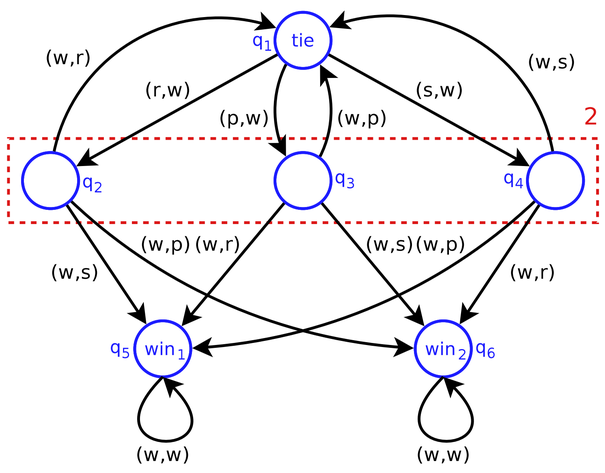
In state q_1, player 1 chooses (r)ock, (p)aper, or (s)cissors, while player 2 has to (w)ait.
Then player 2 makes his choice.

The players can make the game run forever.
⇒ Satisfaction of $\langle\langle 1, 2 \rangle\rangle \Box (\neg win_1 \wedge \neg win_2)$ in q_1.

Coalitional games of imperfect information

Players may not always know "where they are" in the game.

In state q_1, player 1 chooses (r)ock, (p)aper, or (s)cissors, but behind his back.
Player 2 cannot distinguish q_2, q_3, and q_4.



Imperfect information makes the formalization by modal logics more challenging.

What should "A can achieve" mean?

- The players in A have a strategy to achieve their goal (but possibly don't know this).
- The players in A know that they have a strategy to achieve their goal (but possibly don't know what the strategy looks like).
- The players in A have a strategy to achieve their goal, and they know what this strategy looks like.

What should "the players in A know" mean?

- Everyone's knowledge
- Common knowledge
- Distributed knowledge

Perfect vs. imperfect recall:
Are the players assumed to remember the history of the game?

Imperfect recall

Various useful logics in recent literature, for example
Constructive Strategic Logic (CSL):
Formalizes "standard" knowledge + "constructive" knowledge, e.g.,

- $\langle\langle A \rangle\rangle \varphi$ ... A has a strategy to achieve φ,
- $C_A \langle\langle A \rangle\rangle \varphi$ ... it is common knowledge among A that they have a strategy to achieve φ,
- $\mathbb{E}_A \langle\langle A \rangle\rangle \varphi$ ... every player in A can construct a strategy for A to achieve φ.

Perfect recall

Simplest logic: **ATL$_{iR}$:**
Formalizes only one basic strategic concept:
$\langle\langle A \rangle\rangle_{iR} \varphi$ ... every player in A can construct a strategy for A to achieve φ.
Discouraging result (Dima, Ţiplea, February 2011):

The problem
"Is the ATL$_{iR}$ formula φ satisfied in state q of game $\mathcal{G}$?"
is undecidable!

Approximation of ATL$_{iR}$

We introduce two logics ATL$_{i\bar{R}}$ and ATL$_{iR^C}$ which satisfy the implications

$$\langle\langle A \rangle\rangle_{i\bar{R}} \varphi \quad \Rightarrow \quad \langle\langle A \rangle\rangle_{iR} \varphi \quad \Rightarrow \quad \langle\langle A \rangle\rangle_{iR^C} \varphi.$$

These can be used to determine the truth values of ATL$_{iR}$ formulas in some cases, in spite of the undecidability of ATL$_{iR}$.

Idea: Use different assumptions on the players.

- For ATL$_{i\bar{R}}$: Players don't recall the whole history, but *update their knowledge* throughout the game.
- For ATL$_{iR^C}$: Players *communicate* and exchange their knowledge while cooperating.

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